and:

where:

The fluid flow continuity:

Strong form :

BCs:

and at

and at

Function spaces:

Note: missing natural constraints on and .

Apply finite differences:

where

is the timestep length.

Weak form development:

Momentum:

Flow:

Stabilization. Following Hughes (1986) idea for pressure stabilization, we want to add a residual term to increase coercivity.

Assume as scalar and constant, and

We need to select such that we get the coercivity towards , but does not harms the other terms. Let’s work to keep half of the original coercivity.

Peter-Paul:

Invert estimates:

Peter-Paul:

We know that: :

Note: from the inverse estimates, the cosntants are approximately

for linear triangles

for quadratics triangles

Everything back together:

Need to choose and so that

Lets choose:

And and

Now let’s choose so that we keep half the coercivity in the first term:

Solve for and add as a weight:

If we consider the inverse estimates constants similar, they can be absorbed in . If the other ones are not enough to change the game:

Discussion of the results

Let’s open the ratio

For incompressible solids,

If

Which shall be a very small value. (